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$2+1$ KdV(N) equations

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We present some nonlinear partial differential equations in $2+1$-dimensions derived from the KdV equation and its symmetries. We show that all these equations have the same 3-soliton solution structures. The only difference in these solutions are the dispersion relations. We also show that they possess the Painlevé property. © *2011 American Institute of Physics*. [doi:10.1063/1.3629528]

I. INTRODUCTION

After Karasu *et al.*[1] and Kupershmidt's[2] works there have been some attempts to enlarge classes of such integrable nonlinear partial differential equations, known as the hierarchy of KdV(6) equations, and to obtain their $2+1$-dimensional extensions.[3–12] These equations are not in local evolutionary form. Due to this reason their integrability is examined by studying their Painlevé property and by the existence of soliton solutions by Hirota method rather then searching for recursion operators.

Although they differ in some examples,[13] the Painlevé property and the Hirota bilinear approach are powerful tests to examine integrability. In particular, existence of 3-soliton solutions of a nonlinear partial differential equation is believed to be an important indication for the integrability[14–19] (see Ref. 20 for historical development of the integrability). Using this conjecture as a test of integrability, we propose $2+1$-dimensional generalizations of the KdV(N) equations and present their 3-soliton solutions.

In this work we first give these equations in general. Such equations exist not only for KdV family but also for all integrable equations with recursion operators. Concentrating on KdV(N) type of equations, we propose their $2+1$ extensions and give 3-soliton solutions of these proposed equations. We then show that all these equations possess the Painlevé property.

Let $u_t = F[u]$ be a system of integrable nonlinear partial differential equations, where F is a function of $u, u_x, u_{xx}, \cdots$. Let σ_n, $(n = 0, 1, 2, \cdots)$ be infinite number of commuting symmetries. One can write these symmetries as $\sigma_n = R^n\,\sigma_0$, where σ_0 is one of the symmetries of the equation and R is the recursion operator. Then the corresponding evolution equations are given as

$$u_{t_n} = R^n\,\sigma_0, \quad n = 0, 1, 2, \cdots . \tag{1}$$

For $n = 1, 2, 3, \cdots$, Eq. (1) produces hierarchy of the equation $u_t = F[u]$. Since superposition of symmetries is also a symmetry the above equations can be extended to the following more general type:

$$u_{t_{nm}} = a\,R^n\,\sigma_0 + b\,R^m\,\sigma_1, \quad m, n = 0, 1, 2, \cdots , \tag{2}$$

where σ_1 is another symmetry, a and b are arbitrary constants.

Interesting classes of equations are obtained by letting m as a negative integer. As an example, letting $m = -1$ we get

$$u_{t_n} = a\,R^n\,\sigma_0 + b\,R^{-1}(\sigma_1), \quad n = 0, 1, 2, \cdots . \tag{3}$$

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This equation is in evolutionary type but nonlocal, because $R^{-1}(\sigma_1)$ term is an infinite sum of terms containing D^{-1}. It is possible to write (3) as a local differential equation by multiplying both sides by the recursion operator R which is given by

$$R\,[u_{t_n} - a\,R^n\,\sigma_0] = b\,\sigma_1, \quad n = 0, 1, 2, \cdots. \tag{4}$$

For some integrable equations $R(0)$ may not vanish. For example, it is proportional to u_x for the KdV equation. For this reason the constants a and b are introduced for convenience. The above classes of equations (4) are our basic starting point in this work. For a given integrable equation $u_t = F[u]$ it is expected that all the above equations are also integrable. The work of Karasu *et al.*[1] corresponds to $b = 0$, $a = -1$, $n = 1$ for the KdV equation. This equation and its higher order versions KdV$(2n + 4)$, $n = 2, 3, \cdots$ are also integrable.[9] These equations are all in $1 + 1$-dimensions.

II. 2 + 1 KdV(2n + 4) FAMILY AND 3-SOLITON SOLUTIONS

Taking the original equation as the KdV equation, we will investigate 3-soliton solutions of the classes of equations in (4) in $2 + 1$-dimensions corresponding to different values of $(a, b, n, \sigma_0, \sigma_1)$. We conjecture that all these equations are integrable in some sense. Here we examine the integrability by the existence of the 3-soliton solutions. The well-known KdV equation is given as

$$u_t + u_{xxx} + 12\,u\,u_x = 0, \tag{5}$$

with the recursion operator

$$R = D^2 + 8\,u + 4\,u_x\,D^{-1}. \tag{6}$$

For certain values of the set $(a, b, n, \sigma_0, \sigma_1)$ we show that the corresponding $2 + 1$-dimensional equations possess the same 3-soliton solution structures as of the KdV equation and its hierarchy except their dispersion relations.

We obtain $2 + 1$-dimensional equations by assuming $u = u(x, t, y)$ where y is a new independent variable and by letting one of the symmetries $\sigma_0 = u_y$ and $(a = -1, b = 0, n = 1)$. Then we get

$$u_t + u_{xxy} + 8u\,u_y + 4u_x D^{-1}\,u_y = 0. \tag{7}$$

By letting $u = v_x$ we get a local equation as

$$v_{tx} + v_{xxxy} + 8v_x\,v_{xy} + 4v_{xx}\,v_y = 0. \tag{8}$$

This equation has 3-soliton solutions. Let $v = f_x/f$ then

$$f = 1 + f_1 + f_2 + f_3 + A_{12}\,f_1\,f_2 + A_{13}\,f_1\,f_3 + A_{23}\,f_2\,f_3 + A_{123}\,f_1\,f_2\,f_3, \tag{9}$$

where

$$f_i = e^{w_i\,t + k_i\,x + \ell_i\,y + a_i}, \quad (i = 1, 2, 3), \tag{10}$$

$$A_{ij} = \left(\frac{k_i - k_j}{k_i + k_j}\right)^2, \quad (i, j = 1, 2, 3), \tag{11}$$

$$A_{123} = A_{12}\,A_{13}\,A_{23}. \tag{12}$$

Here $w_i, k_i, \ell_i, a_i (i = 1, 2, 3)$ are arbitrary constants. Dispersion relations are

$$w_i = -\ell_i\,k_i^2, \quad (i = 1, 2, 3). \tag{13}$$

It is possible to show that all KdV$(2n + 4)$, $(n = 2, 3, \cdots)$ equations $R\,(u_t + R^n\,u_y) = 0$ have 3-soliton solutions same as the 3-soliton solutions of the KdV hierarchy in (1).

Below we give a different class of $2+1$-dimensional equations (4) by letting $\sigma_1 = u_y, \sigma_0 = u_x$, and $a = b = -1$.

A. ($n = 1$) 2 + 1 KdV(6) equation

Equation (4) reduces to $R\,(u_t + R\,u_x) + u_y = 0$. Letting $u = v_x$ we get

$$v_t + v_{xxx} + 6v_x^2 = q, \tag{14}$$

$$v_{xy} + q_{xxx} + 8v_x\,q_x + 4v_{xx}\,q = 0. \tag{15}$$

Dispersion relations are

$$w_i = -k_i^3 - \frac{\ell_i}{k_i^2}, \quad i = 1, 2, 3. \tag{16}$$

B. ($n = 2$) 2 + 1 KdV(8) equation

Equation (4) reduces to $R\,(u_t + R^2\,u_x) + u_y = 0$. Letting $u = v_x$ we get

$$v_t + v_{xxxxx} + 20\,v_x\,v_{xxx} + 10\,v_{xx}^2 + 40\,v_x^3 = q, \tag{17}$$

$$v_{xy} + q_{xxx} + 8v_x\,q_x + 4v_{xx}\,q = 0. \tag{18}$$

Dispersion relations are

$$w_i = -k_i^5 - \frac{\ell_i}{k_i^2}, \quad i = 1, 2, 3. \tag{19}$$

C. ($n = 3$) 2 + 1 KdV(10) equation

Equation (4) reduces to $R\,(u_t + R^3\,u_x) + u_y = 0$. Letting $u = v_x$ we get

$$\begin{aligned} v_t + v_{7x} + 42\,v_{3x}^2 + 280\,v_x^4 + 56\,v_{2x}\,v_{4x} + 280\,v_x\,v_{2x}^2 \\ +28\,v_x\,v_{5x} + 280\,v_x^2\,v_{3x} = q, \end{aligned} \tag{20}$$

$$v_{xy} + q_{xxx} + 8v_x\,q_x + 4v_{xx}\,q = 0. \tag{21}$$

Dispersion relations are

$$w_i = -k_i^7 - \frac{\ell_i}{k_i^2}, \quad i = 1, 2, 3. \tag{22}$$

In addition to above equations we conjecture that 3-soliton solutions of the $2+1$ KdV($2n+4$) equation exists for all $n \geq 4$ and they are given as

D. ($n \geq 4$) 2 + 1 KdV($2n + 4$) equation

For all values of $n \geq 4$ we have

$$u_t + R^n\,u_x = q, \tag{23}$$

$$v_{xy} + q_{xxx} + 8v_x q_x + 4v_{xx} q = 0. \tag{24}$$

Here $u = v_x$. Dispersion relations are

$$w_i = -k_i^{2n+1} - \frac{\ell_i}{k_i^2}, \quad i = 1, 2, 3. \tag{25}$$

III. PAINLEVÉ PROPERTY OF 2 + 1 KdV(2n + 4) EQUATIONS FOR N = 1, 2, 3

In this section, we check the Painlevé property of the $2 + 1$ KdV$(2n + 4)$ equations for $n = 1, 2, 3$. We used a MAPLE package called PDEPtest[21] for this purpose. Here the WTC-Kruskal algorithm is used.[22–24] Note that a nonlinear partial differential equation is said to possess the Painlevé property, if all solutions of it can be expressed as Laurent series

$$v^{(i)}(x, t, y) = \sum_{j=0}^{\infty} v_j^{(i)}(x, t, y)\phi(x, t, y)^{(j+\alpha_i)}, \quad i = 1, \ldots, m, \tag{26}$$

with sufficient number of arbitrary functions as the order of the equation, $v_j^{(i)}(x, t, y)$ are analytic functions, α_i are negative integers.

A. 2 + 1 KdV(6) equation

By leading order analysis, we see that $2 + 1$ KdV(6) equation admits two branches. The leading exponents for these two branches are -1, and the leading order coefficients are

$$\text{(i)} \quad v_0 = \phi_x \qquad , \qquad \text{(ii)} \quad v_0 = 3\phi_x.$$

The corresponding truncated expansions for these two branches are

$$\text{(i)} \quad v = \frac{\phi_x}{\phi} + v_1 \qquad , \qquad \text{(ii)} \quad v = \frac{3\phi_x}{\phi} + v_1.$$

The resonances of the above branches are

$$\text{(i)} \quad r = -1, 1, 2, 5, 6, 8 \qquad , \qquad \text{(ii)} \quad r = -1, 1, -3, 6, 8, 10.$$

It is clear that branch (i) is the principal (generic) one and the other one is secondary(non-generic) branch. For the principal branch, the coefficients of the series (26) at non-resonances are

$$\begin{aligned}
&v_0 = 1, \quad v_3 = 0, \\
&v_4 = -\frac{1}{10} v_2 \psi_t + v_2^2 - \frac{1}{120}\psi_y + \frac{1}{30} v_{1_t}, \\
&v_7 = -v_2 v_5 + \frac{1}{20} v_5 \psi_t + \frac{1}{480} v_{2_y} + \frac{1}{480} v_{2_t} \psi_t + \frac{1}{480} v_2 \psi_{tt} \\
&\qquad - \frac{1}{40} v_2 v_{2_t} + \frac{1}{5760}\psi_{yt} - \frac{1}{1440} v_{1_{tt}},
\end{aligned}$$

where v_1, v_2, v_5, v_6, v_8 are arbitrary functions of the variables y and t and $\phi(x, t, y) = x - \psi(t, y)$. For the second branch, the coefficients of (26) at non-resonances are

$$\begin{aligned}
&v_0 = 3, \quad v_2 = \frac{1}{20}\psi_t,\\
&v_4 = -\frac{1}{2800}\psi_t^2 + \frac{1}{120}v_{1_t} - \frac{1}{840}\psi_y,\\
&v_5 = 0, \quad v_7 = -\frac{1}{28800}\psi_t\psi_{tt} - \frac{1}{14400}\psi_{yt} + \frac{1}{7200}v_{1_{tt}},\\
&v_9 = -\frac{1}{806400}\psi_t\psi_{yt} + \frac{1}{201600}v_{1_{yt}} - \frac{1}{806400}\psi_{yy} - \frac{1}{80}v_{6_t}\\
&\quad -\frac{1}{1680000}\psi_t^2\psi_{tt} + \frac{1}{504000}\psi_t v_{1_{tt}} + \frac{1}{252000}\psi_{tt}v_{1_t} - \frac{1}{1008000}\psi_{tt}\psi_y,
\end{aligned}$$

where v_1, v_6, v_8, v_{10} are arbitrary functions of the variables y and t and $\phi(x, t, y) = x - \psi(t, y)$.

B. 2 + 1 KdV(8) equation

For the 2 + 1 KdV(8) equation, we get the following information from the Painlevé property. By leading order analysis, we see that 2 + 1 KdV(8) equation admits three branches. The leading exponents for these three branches are −1, and the leading order coefficients are

$$\text{(i)} \quad v_0 = \phi_x \quad , \quad \text{(ii)} \quad v_0 = 3\phi_x \quad , \quad \text{(iii)} \quad v_0 = 6\phi_x.$$

The corresponding truncated expansions for these three branches are

$$\text{(i)} \quad v = \frac{\phi_x}{\phi} + v_1 \quad , \quad \text{(ii)} \quad v = \frac{3\phi_x}{\phi} + v_1 \quad , \quad \text{(iii)} \quad v = \frac{6\phi_x}{\phi} + v_1.$$

The resonances of the above branches are

$$\text{(i)} \quad r = -1, 1, 2, 4, 5, 7, 8, 10 \quad , \quad \text{(ii)} \quad r = -1, 1, 2, -3, 7, 8, 10, 12 \quad ,$$

$$\text{(iii)} \quad r = -1, 1, -3, -5, 8, 10, 12, 14.$$

Obviously, branch (i) is the principal one and the other two are secondary branches. For the principal branch, the coefficients of the series (26) at non-resonances are

$$\begin{aligned}
&v_0 = 1, \quad v_3 = 0,\\
&v_6 = -3v_2v_4 - \frac{3}{280}v_2\psi_t + \frac{1}{280}v_{1_t} - \frac{1}{1120}\psi_y + v_2^3,\\
&v_9 = -\frac{1}{2240}v_{4_t} + \frac{3}{2800}v_5\psi_t + \frac{1}{22400}v_{2_y} - \frac{3}{5}v_2v_7\\
&\quad -\frac{3}{10}v_4v_5 - \frac{3}{10}v_2^2v_5 + \frac{1}{2800}v_2v_{2_t},
\end{aligned}$$

where $v_1, v_2, v_4, v_5, v_7, v_8, v_{10}$ are arbitrary functions of the variables y and t and $\phi(x, t, y) = x - \psi(t, y)$.

For the second branch, the coefficients of (26) at non-resonances are

$$\begin{aligned}
v_0 &= 3, \quad v_3 = 0,\\
v_4 &= \frac{1}{3}v_2^2 - \frac{1}{840}\psi_t, \quad v_5 = 0,\\
v_6 &= \frac{2}{3}v_2^3 - \frac{1}{2520}v_{1_t} - \frac{1}{630}v_2\psi_t + \frac{1}{10080}\psi_y,\\
v_9 &= -v_2v_7 + \frac{1}{2016}v_2v_{2_t} - \frac{1}{806400}\psi_{tt} - \frac{1}{40320}v_{2_y},\\
v_{11} &= \frac{1}{2520}v_7\psi_t + \frac{1}{3}v_2^2v_7 + \frac{1}{45360}v_2v_{2_y} - \frac{1}{25401600}\psi_{ty}\\
&\quad -\frac{1}{2268}v_2^2v_{2_t} + \frac{1}{1411200}v_2\psi_{tt} + \frac{1}{12700800}v_{1_{tt}},
\end{aligned}$$

where $v_1, v_2, v_7, v_8, v_{10}, v_{12}$ are arbitrary functions of the variables y and t and $\phi(x, t, y) = x - \psi(t, y)$.

For the third branch, the coefficients of (26) at non-resonances are

$$\begin{aligned}
v_0 &= 6, \quad v_2 = 0, \quad v_3 = 0, \quad v_4 = -\frac{1}{2520}\psi_t,\\
v_6 &= \frac{1}{110880}\psi_y - \frac{1}{27720}v_{1_t}, \quad v_7 = 0,\\
v_9 &= \frac{1}{5644800}\psi_{tt}, \quad v_{11} = \frac{1}{101606400}\psi_{ty} - \frac{1}{50803200}v_{1_{tt}},\\
v_{13} &= \frac{29}{384072192000}\psi_{tt}\psi_t + \frac{1}{16094453760}\psi_{yy} - \frac{1}{4023613440}v_{1_{ty}}\\
&\quad -\frac{13}{120960}v_{8_t},
\end{aligned}$$

where $v_1, v_8, v_{10}, v_{12}, v_{14}$ are arbitrary functions of the variables y and t and $\phi(x, t, y) = x - \psi(t, y)$.

C. 2 + 1 KdV(10) equation

By leading order analysis, we see that $2+1$ KdV(10) equation admits four branches. The leading exponents for these four branches are -1, and the leading order coefficients are

$$\text{(i)} \quad v_0 = \phi_x \quad , \quad \text{(ii)} \quad v_0 = 3\phi_x \quad ,$$

$$\text{(iii)} \quad v_0 = 6\phi_x \quad , \quad \text{(iv)} v_0 = 10\phi_x.$$

The corresponding truncated expansions for these four branches are

$$\text{(i)} \quad v = \frac{\phi_x}{\phi} + v_1 \quad , \quad \text{(ii)} \quad v = \frac{3\phi_x}{\phi} + v_1 \quad ,$$

$$\text{(iii)} \quad v = \frac{6\phi_x}{\phi} + v_1 \quad , \quad \text{(iv)} \quad v = \frac{10\phi_x}{\phi} + v_1.$$

The resonances of the above branches are

$$\begin{aligned}
&\text{(i)} \quad r = -1, 1, 2, 4, 5, 6, 7, 9, 10, 12,\\
&\text{(ii)} \quad r = -1, 1, 2, -3, 4, 7, 9, 10, 12, 14,\\
&\text{(iii)} \quad r = -1, 1, 2, -3, -5, 9, 10, 12, 14, 16,\\
&\text{(iv)} \quad r = -1, 1, -3, -5, -7, 10, 12, 14, 16, 18.
\end{aligned}$$

The branch (i) is the principal one and the other three are secondary branches. For the principal branch, the coefficients of the series (26) at non-resonances are

$$
\begin{aligned}
v_0 &= 1, \quad v_3 = 0,\\
v_8 &= \frac{1}{6}v_4^2 - \frac{5}{3}v_2^2 v_4 - \frac{1}{3024}v_2\psi_t - \frac{10}{9}v_2 v_6 - \frac{1}{36288}\psi_y\\
&\quad + \frac{5}{18}v_2^4 + \frac{1}{9072}v_{1_t},\\
v_{11} &= \frac{1}{1814400}v_{2_y} - \frac{1}{181440}v_{4_t} - \frac{16}{45}v_5 v_6 + \frac{1}{226800}v_2 v_{2_t}\\
&\quad - \frac{2}{15}v_2^2 v_7 - \frac{1}{3}v_4 v_7 + \frac{1}{75600}v_5\psi_t - \frac{2}{45}v_2^3 v_5 - \frac{2}{15}v_2 v_4 v_5 - \frac{4}{9}v_2 v_9,
\end{aligned}
$$

where $v_1, v_2, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{12}$ are arbitrary functions of the variables y and t and $\phi(x, t, y) = x - \psi(t, y)$.

For the second branch, the coefficients of (26) at non-resonances are

$$
\begin{aligned}
v_0 &= 3, \quad v_3 = 0, \quad v_5 = 0, \quad v_6 = \frac{1}{10080}\psi_t + 3v_4 v_2 - \frac{1}{3}v_2^3,\\
v_8 &= \frac{1}{133056}\psi_y - 5v_2^2 v_4 - \frac{1}{33264}v_{1_t} - \frac{11}{2}v_4^2 - \frac{1}{5544}v_2\psi_t + \frac{5}{6}v_2^4,\\
v_{11} &= \frac{1}{7}v_4 v_7 - \frac{1}{2540160}v_{2_y} - \frac{1}{317520}v_2 v_{2_t} + \frac{1}{60480}v_{4_t} - \frac{2}{7}v_2^2 v_7 - \frac{4}{7}v_2 v_9,\\
v_{13} &= -\frac{1}{5364817920}\psi_{tt} - \frac{1}{99792}v_2 v_{4_t} - \frac{5}{1596672}v_4 v_{2_t} + \frac{1}{338688}v_2^2 v_{2_t}\\
&\quad - \frac{1}{6}v_4 v_9 - \frac{1}{33264}v_7\psi_t + \frac{25}{126}v_2^3 v_7 + \frac{5}{42}v_2^2 v_9 - \frac{15}{14}v_2 v_4 v_7\\
&\quad + \frac{1}{6386688}v_{4_y} + \frac{1}{7451136}v_2 v_{2_y},
\end{aligned}
$$

where $v_1, v_2, v_4, v_7, v_9, v_{10}, v_{12}, v_{14}$ are arbitrary functions of the variables y and t and $\phi(x, t, y) = x - \psi(t, y)$.

For the third branch, the coefficients of (26) at non-resonances are

$$
\begin{aligned}
v_0 &= 6, \quad v_3 = 0, \quad v_4 = \frac{1}{5}v_2^2, \quad v_5 = 0,\\
v_6 &= \frac{2}{15}v_2^3 + \frac{1}{110880}\psi_t, \quad v_7 = 0,\\
v_8 &= \frac{23}{75}v_2^4 + \frac{1}{432432}v_{1_t} + \frac{1}{72072}v_2\psi_t - \frac{1}{1729728}\psi_y,\\
v_{11} &= \frac{1}{7257600}v_{2_y} - \frac{1}{259200}v_2 v_{2_t} - v_2 v_9,\\
v_{13} &= \frac{1}{285120}v_2^2 v_{2_t} + \frac{1}{16094453760}\psi_{tt} + \frac{2}{5}v_2^2 v_9 - \frac{1}{7983360}v_2 v_{2_y},\\
v_{15} &= -\frac{1}{432432}v_9\psi_t - \frac{1}{9}v_2^3 v_9 - \frac{47}{37065600}v_2^3 v_{2_t} - \frac{1}{523069747200}v_{1_{tt}}\\
&\quad - \frac{1}{40236134400}v_2\psi_{tt} + \frac{1}{1046139494400}\psi_{ty} + \frac{47}{1037836800}v_2^2 v_{2_y},
\end{aligned}
$$

where $v_1, v_2, v_9, v_{10}, v_{12}, v_{14}, v_{16}$ are arbitrary functions of the variables y and t and $\phi(x, t, y) = x - \psi(t, y)$.

For the fourth branch, the coefficients of (26) at non-resonances are

$$v_0 = 10, \quad v_2 = 0, \quad v_3 = 0, \quad v_4 = 0, \quad v_5 = 0,$$
$$v_6 = \frac{1}{480480}\psi_t, \quad v_7 = 0,$$
$$v_8 = -\frac{1}{25945920}\psi_y + \frac{1}{6486480}v_{1_t}, \quad v_9 = 0,$$
$$v_{11} = 0, \quad v_{13} = -\frac{1}{80472268800}\psi_{tt},$$
$$v_{15} = -\frac{1}{3835844812800}\psi_{ty} + \frac{1}{1917922406400}v_{1_{tt}},$$
$$v_{17} = -\frac{1}{1183632113664000}\psi_{yy} + \frac{1}{295908028416000}v_{1_{ty}} - \frac{17}{39916800}v_{10_t},$$

where $v_1, v_{10}, v_{12}, v_{14}, v_{16}, v_{18}$ are arbitrary functions of the variables y and t and $\phi(x, t, y) = x - \psi(t, y)$.

To sum up, the principal branches of $2 + 1$ KdV($2n + 4$) equations for $n = 1, 2, 3$ admit arbitrary functions and the compatibility conditions at all non-negative integer resonances are satisfied identically. Hence $2 + 1$ KdV($2n + 4$) equations for $n = 1, 2, 3$ possess the Painlevé property.

IV. CONCLUSION

We introduced a new class of nonlinear partial differential equations, $2 + 1$ KdV($2n + 4$) equations, in $2 + 1$ dimensions derived from the KdV equation and its symmetries. We have given 3-soliton solutions of these equations for $n = 1, 2, 3$. We showed that they also have the Painlevé property for $n = 1, 2, 3$. We conjecture that these equations have 3-soliton solutions and possess the Painlevé property for all positive integer n.

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